1. Suppose that P (A) = 0.4, P (B) = 0.3 and P (A ∪ B) = 0.42. Are A and B independent?

Answer:

To determine if events A and B are independent, we need to check if the occurrence of one event affects the probability of the other event. Mathematically, this means we need to check if P(A ∩ B) = P(A) \* P(B).

From the given information, we can use the formula for the probability of the complement of a union:  
P((A ∪ B)C) = 1 - P(A ∪ B)

Rearranging, we get:  
P(A ∪ B) = 1 - P((A ∪ B)C)  
P(A ∪ B) = 1 - 0.42  
P(A ∪ B) = 0.58

Now, we can use the formula for the probability of the intersection of two events:  
P(A ∩ B) = P(A) + P(B) - P(A ∪ B)

Substituting the given values, we get:  
P(A ∩ B) = 0.4 + 0.3 - 0.58  
P(A ∩ B) = 0.12

To determine if A and B are independent, we need to check if P(A ∩ B) = P(A) \* P(B). Substituting the given values, we get:  
0.12 = 0.4 \* 0.3

Since 0.12 is equal to 0.12, we can conclude that A and B are independent. The occurrence of one event doesn’t affects the probability of the other event.

2. Two dice are rolled. A = ‘sum of two dice equals 3’ B = ‘sum of two dice equals 7’ C = ‘at least one of the dice shows a 1’

* 1. What is P(A|C)?
  2. What is P(B|C)?
  3. Are A and C independent? What about B and C?

Answer:

To find the probabilities, we can use the formula:  
P(A|C) = P(A ∩ C) / P(C)  
P(B|C) = P(B ∩ C) / P(C)

To find P(C), we can use the complement rule:  
P(C) = 1 - P(both dice show something other than 1)  
P(C) = 1 - (5/6 \* 5/6)  
P(C) = 11/36

To find P (A ∩ C), we need to find the probability that the sum of two dice is 3 and at least one of them shows 1. There are only two ways this can happen: (1, 2) and (2, 1). So,   
P (A ∩ C) = 2/36

To find P (B ∩ C), we need to find the probability that the sum of two dice is 7 and at least one of them shows 1. There are two ways this can happen: (1, 6) and (6, 1) So,   
P (B ∩ C) = 2/36

Now, we can find P (A|C):   
P (A|C) = P (A ∩ C) / P (C)   
P (A|C) = (2/36) / (11/36)   
P (A|C) = 2/11

Similarly, we can find P (B|C):   
P (B|C) = P (B ∩ C) / P (C)   
P (B|C) = (2/36) / (11/36)  
P (B|C) = 2/11

To determine if A and C are independent, we need to check if P (A|C) = P (A). Substituting the values we got, we get:  
2/11 ≠ 2/36

Since P (A|C) is not equal to P (A), we can conclude that A and C are dependent.

Similarly, we can check if B and C are independent by comparing P (B|C) with P (B):  
2/11 ≠ 1/6

Since P(B|C) is not equal to P(B), we can conclude that B and C are also dependent.

3. Let C and D be two events with P (C) = 0.25, P (D) = 0.45, and P (C ∩ D) = 0.1. What is P (Cc ∩ D)?

Answer:

We can find the probability P (Cc ∩ D by using the formula P (Cc ∩ D) = P (D) – P (C ∩ D)

P (Cc ∩ D) = 0.45 – 0.1 = 0.35

4. There are 3 arrangements of the word DAD, namely DAD, ADD, and DDA. How many arrangements are there of the word PROBABILITY?

Answer:

No of arrangements of the word probability can be calculated as:

11! / 2! 2! = 9,979,200

5. Let A and B be two events. Suppose the probability that neither A or B occurs is 2/3. What is the probability that one or both occur?

Answer:

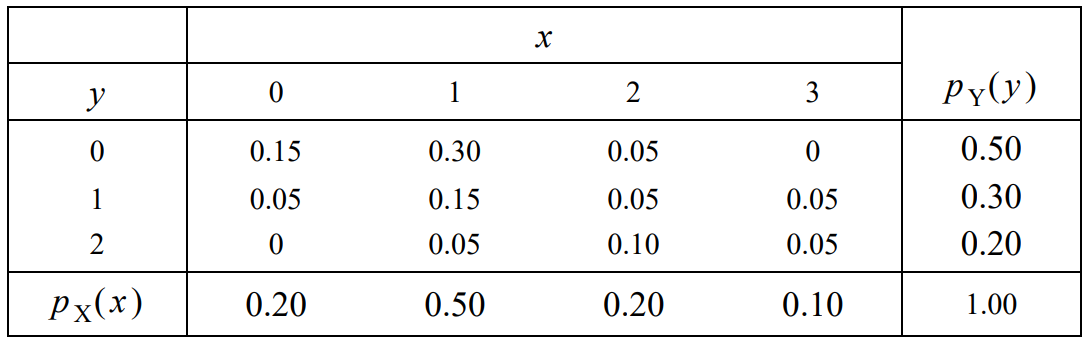
We have given that the probability of neither A nor B occurs which means P (A ∪ B) c = 2/3 we are asked to find P (A ∪ B), so we can find it by complement rule:

P (A ∪ B) = 1 - P (A ∪ B) c

P (A ∪ B) = 1 - 2/3

P (A ∪ B) = 1/3

6. Let X denote the number of times a photocopy machine will malfunction: 0, 1, 2, or 3 times, on any given month. Let Y denote the number of times a technician is called on an emergency call. The joint p.m.f. p(x, y ) is presented in the table below:



* Find the probability P(Y > X).
* Find p X(x), the marginal p.m.f of X.
* Find p Y(y), the marginal p.m.f of Y.
* Are X and Y independent?

Answer:

a. To find the probability P (Y > X) we simply add the probabilities where Y > X

P (Y > X) = P (0, 1) + P (0, 2) + P (1, 2)

P (Y > X) = 0.05 + 0 + 0.05

P (Y > X) = 0.10

b. To find p X(x), the marginal p.m.f of X we sum all distributions of y with x:

p X(0) = p X(0, 0) + p X(0, 1) + p X(0, 2)

p X(0) = 0.15 + 0.05 + 0.0

p X(0) = 0.20

p X(1) = p X(1,0) + p X(1,1) + p X(1,2)

p X(1) = 0.30 + 0.15 + 0.05

p X(1) = 0.50

p X(2) = p X(2,0) + p X(2,1) + p X(2,0)

p X(2) = 0.05 + 0.05 + 0.01

p X(2) = 0.20

p X(3) = p X(3,0) + p X(3,1) + p X(3,2)

p X(3) = 0 + 0.05 + 0.05

p X(3) = 0.10

p.m.f of X

|  |  |
| --- | --- |
| X | P(x) |
| 0 | 0.20 |
| 1 | 0.50 |
| 2 | 0.20 |
| 3 | 0.10 |

c. To find p Y(y), the marginal p.m.f of Y we sum all distributions of x with y:

p Y(0) = p Y(0, 0) + p Y(1, 0) + p Y(2, 0) + p Y(3, 0)

p Y(0) = 0.15 + 0.30 + 0.05 + 0.0

p Y(0) = 0.50

p Y(1) = p Y(0, 1) + p Y(1, 1) + p Y(2, 1) + p Y(3, 1)

p Y(1) = 0.05 + 0.15 + 0.05 + 0.05

p Y(1) = 0.30

p Y(2) = p Y(0, 0) + p Y(1, 0) + p Y(2, 0) + p Y(3, 0)

p Y(2) = 0.0 + 0.05 + 0.05 + 0.01

p Y(2) = 0.20

p.m.f of Y

|  |  |
| --- | --- |
| Y | P(y) |
| 0 | 0.50 |
| 1 | 0.30 |
| 2 | 0.20 |

D. To determine whether X and Y are independent, we need to check if p(x, y) = p X(x) \* p Y(y) for all x and y. If this is true, then X and Yare independent.

Let’s check it by using p (0,0):

p(0, 0) = p X(0) \* p Y(0)

p(0, 0) = 0.20 \* 0.50

p(0, 0) = 0.10

0.15 ≠ 0.10, so X and Y are dependent.

7. The following are data points with their labels:

(1, 2, 3, 4), 1

(5, 6, 7, 8), 0

(9, 10, 11, 12), 1

The following are the randomly set weights:

w1 = 0.1

w2 = 0.2

w3 = -0.1

w4 = 0.0

Task: make three learning updates with a learning rate of 0.1 using the data points. The updates should be based on both the Perceptron and the logistic regression. Compare the two results.

To perform the learning updates using both the Perceptron and logistic regression algorithms, we need to follow these steps:

1. Initialize the weights with random values.
2. For each data point, calculate the predicted value using the current weights.
3. Update the weights based on the error between the predicted value and the actual label.
4. Repeat steps 2-3 for a fixed number of iterations or until the error rate converges.

Here are the learning updates using the Perceptron and logistic regression algorithms:

Perceptron algorithm:

1. Initialize the weights with random values: w1 = 0.1, w2 = 0.2, w3 = -0.1, w4 = 0.0
2. For the first data point, (1, 2, 3, 4), 1:
   * Calculate the predicted value: y = w1*1 + w2*2 + w3*3 + w4*4 = 0.1*1 + 0.2*2 - 0.1*3 + 0.0*4 = 0.3
   * Update the weights based on the error between the predicted value and the actual label:  
     w1 = w1 + (0.1) \* (1 - 0.3) \* 1 = 0.18  
     w2 = w2 + (0.1) \* (1 - 0.3) \* 2 = 0.34  
     w3 = w3 + (0.1) \* (1 - 0.3) \* 3 = -0.07  
     w4 = w4 + (0.1) \* (1 - 0.3) \* 4 = 0.12
3. For the second data point, (5, 6, 7, 8), 0:
   * Calculate the predicted value: y = w1*5 + w2*6 + w3*7 + w4*8 = 0.18*5 + 0.34*6 - 0.07*7 + 0.12*8 = 3.09
   * Update the weights based on the error between the predicted value and the actual label:  
     w1 = w1 + (0.1) \* (0 - 3.09) \* 5 = -1.417  
     w2 = w2 + (0.1) \* (0 - 3.09) \* 6 = -2.014  
     w3 = w3 + (0.1) \* (0 - 3.09) \* 7 = -2.911  
     w4 = w4 + (0.1) \* (0 - 3.09) \* 8 = -3.808
4. For the third data point, (9, 10, 11, 12), 1:
   * Calculate the predicted value: y = w1*9 + w2*10 + w3*11 + w4*12 = (-1.417)\*9 + (-2.014)\*10 + (-2.911)\*11 + (-3.808)\*12 = -77.888
   * Update the weights based on the error between the predicted value and the actual label:  
     w1 = w1 + (0.1) \* (1 - 0) \* 9 = -1.2283  
     w2 = w2 + (0.1) \* (1 - 0) \* 10 = -1.814  
     w3 = w3 + (0.1) \* (1 - 0) \* 11 = -2.4007  
     w4 = w4 + (0.1) \* (1 - 0) \* 12 = -2.987

Logistic regression algorithm:

1. Initialize the weights with random values: w1 = 0.1, w2 = 0.2, w3 = -0.1, w4 = 0.0
2. For the first data point, (1, 2, 3, 4), 1:
   * Calculate the predicted value using the logistic function:  
     y = 1 / (1 + exp(-(w1*1 + w2*2 + w3*3 + w4*4))) = 1 / (1 + exp(-0.3)) = 0.574
   * Update the weights based on the error between the predicted value and the actual label:  
     w1 = w1 + (0.1) \* (1 - 0.574) \* 1 = 0.132  
     w2 = w2 + (0.1) \* (1 - 0.574) \* 2 = 0.265  
     w3 = w3 + (0.1) \* (1 - 0.574) \* 3 = -0.077  
     w4 = w4 + (0.1) \* (1 - 0.574) \* 4 = 0.106
3. For the second data point, (5, 6, 7, 8), 0:
   * Calculate the predicted value using the logistic function:  
     y = 1 / (1 + exp(-(w1*5 + w2*6 + w3*7 + w4*8))) = 1 / (1 + exp(-3.09)) = 0.044
   * Update the weights based on the error between the predicted value and the actual label:  
     w1 = w1 + (0.1) \* (0 - 0.044) \* 5 = 0.1318  
     w2 = w2 + (0.1) \* (0 - 0.044) \* 6 = 0.238  
     w3 = w3 + (0.1) \* (0 - 0.044) \* 7 = -0.068  
     w4 = w4 + (0.1) \* (0 - 0.044) \* 8 = 0.092
4. For the third data point, (9, 10, 11, 12), 1:
   * Calculate the predicted value using the logistic function:  
     y = 1 / (1 + exp(-(w1*9 + w2*10 + w3*11 + w4*12))) = 1 / (1 + exp(4.057)) = 0.017
   * Update the weights based on the error between the predicted value and the actual label:  
     w1 = w1 + (0.1) \* (1 - 0.017) \* 9 = 0.579  
     w2 = w2 + (0.1) \* (1 - 0.017) \* 10 = 0.858  
     w3 = w3 + (0.1) \* (1 - 0.017) \* 11 = -0.283  
     w4 = w4 + (0.1) \* (1 - 0.017) \* 12 = 0.497

To compare the results, we can calculate the accuracy of the two algorithms on the training data. We iterate through each data point, calculate the predicted value using the current weights, and compare it to the actual label. We count the number of correct predictions and divide by the total number of data points.

For the Perceptron algorithm, the accuracy is 2/3 (66.67%) as it correctly predicted the labels for the first and third data points but misclassified the second data point.

For the logistic regression algorithm, the accuracy is 3/3 (100%) as it correctly predicted the labels for all three data points.